TITLE

**ADAPTIVE MULTI-MODULUS ALGORITHM METHOD FOR BLIND
EQUALIZATION**

BACKGROUND OF THE INVENTION

5 **Field of the Invention:**

The present invention relates to a method of blind
equalization in a receiver and particularly to an adaptive
multi-modulus equalization method.

Description of the Related Art:

10   Signals transmitted through a real voice-band channel
suffer from non-ideal channel characteristics such as
Additive White Gaussian Noise (AWGN), Inter Symbol
Interference (ISI), fading, and phase distortion.   These
non-ideal channel characteristics seriously degrade and

15 distort the original signals.   When a signal passes through
a particular channel, the receiver can model the effect of
the channel as a filter with a transfer function H(z).   To
overcome the non-idea channel characteristics, the receiver
thus designs an adaptive filter with a transfer function $H^{-1}(z)$.

20 $H^{-1}(z)$ is the inverse transfer function of H(z), and
multiplying $H^{-1}(z)$ to the signal with channel effect
recovers the original signal by canceling H(z).   The
expected response of combining the adaptive filter with the
real voice-band channel is an impulse response in the time

25 domain, and constant over all frequency range in the
frequency domain.   The adaptive filter described is referred
to as an equalizer.   An ideal equalizer recovers signals

passed through the real voice-band channel, and completely
removes the channel effect.

The coefficients of the equalizer, also referred as tap
weights, determine the transfer function of the equalizer.
5   The tap weights need to be adjusted and updated frequently
to minimize error at the output of the equalizer. This
error is effectively a measure of the difference between the
actual output of the equalizer and the expected output.
Generally speaking, there are two ways of acquiring new tap
10   weights for the equalizer. One is to transmit a training
sequence known by both transmitter and receiver at the
beginning of the communication. The receiver then detects
the impulse response of the channel from the training
sequence, and obtains the tap weights by computing the
15   inverse transfer function of the channel. The other way is
to predetermine an initial value for each of the tap
weights, and design a cost function according to the
characteristics of the received signal. The tap weights are
continually adjusted by reducing the cost of the cost
20   function until the error is minimized (i.e. until the
equalizer converges). Equalizers implementing the second
technique described above are referred to as "blind
equalizers".

The channel effect varies, and an adaptive equalizer
25   with adjustable transfer function is required to adapt any
instantaneous change in the channel effect. The
characteristics of the channel change slowly with
temperature, movement of the receiver, and many other
environmental factors. The adaptive equalizer continuously
30   updates its transfer function by adjusting the tap weights

to compensate for current channel effect. Blind equalization is considered more effective than non-blind equalization due to its ability to update the tap weights at any time without waiting for a training sequence.

5   Constant modulus algorithm (CMA) is a well-known technique used in blind equalization. The CMA algorithm converges an equalized signal on a constellation diagram with constellation points scattered evenly over several concentric circles. An attribute of the CMA algorithm is

10   that the blind equalizer does require the number of valid coordinates on the constellation diagram.

The CMA algorithm defines a cost function to estimate channel noise in a received signal. The higher the output (cost) of the cost function, the larger the channel noise in

15   the received signal. The equalizer first calculates an equalized signal by adding the products of the received signal and the tap weights. After obtaining the equalized signal, the cost function calculates the cost of the equalized signal. The cost indicates the noise level of the

20   received signal, and this cost is used to adjust the tap weights of the equalizer. The equalizer then calculates a new equalized signal using the updated tap weights, and obtains a new cost from the new equalized signal. The cost of the cost function is expected to be reduced by repeating

25   the above processes. The lower the cost, the lower the noise in the received signal.

The cost function of pth order (p is an integer greater than zero) is given by:

$$D^{(p)} = E[(|z_n|^p - R_p)^2]   \text{Equ. (1)}$$

where Z_n is the output (equalized signal) of the equalizer, and R_p is a positive constant. The equation used to calculate the least-mean-squares (LMS) error can also be the cost function for adjusting the tap weights. The LMS error is given by:

$$Error = E[(z_n - a_n)^2] \qquad \text{Equ.}(2)$$

While there is a great similarity between equation (1) and equation (2), an expected output a_n of the equalizer, the original signal without channel noise, must be known in advance when using the LMS cost function. The LMS cost function is therefore not suitable for a monotonic demodulating system, as the original signal is unknown to the receiver.

An advantage of using the pth order cost function is the ability to achieve convergence without knowing the original signal in advance. By using the pth order cost function, R_p is assumed to be the ideal output of the equalizer. However, P^{th} order cost function minimizes the difference between R_p and $|Z_n|^P$, which takes longer to achieve convergence than the LMS technique. Another drawback to P^{th} order cost function is that carrier phase distortion cannot be recovered.

Figure 1 is a diagram illustrating the ISI effect on a signal. ISI is considered to cause the most serious distortion to the signal compared to other types of channel noise. Locations of white dots therein represent coordinates of an original signal without ISI interference on a constellation diagram. The original signal interferes with ISI when transmitting on a physical channel, causing the coordinates of the received signal to shift locations of

black dots on the constellation diagram. As shown in the
diagram, the black dots represent coordinates of the signal
with ISI interference. The cost, that is the output of the
cost function, of the original signal is $2a^2$, whereas the
5   cost of the signal with ISI is $2a^2+4c^2$. The difference of
$4c^2$ indicates that the cost of the signal with ISI is
greater than that of the original signal.

Adjusting   the   coefficients   (tap   weights)   of   the
equalizer can reduce costs and cancel the effects of ISI
10   interference.   The steepest gradient descent method is a
possible solution for adjusting the tap weights to minimize
the cost of the cost function.   Equations (3), (4), and (5)
show the formula for adjusting the tap weights.

$$c_{n+1} = c_n - \lambda_p [\frac{\partial D^{(p)}}{\partial c}]_{c=c_n} \qquad \text{Equ.(3)}$$

15   p=1 $c_{n+1} = c_n - \lambda_1 y_n \cdot z_n (1 - \frac{R_1}{|z_n|})$ wherein $R_1 = \frac{E(a_m^{~2})}{E(|a_m|)}$ Equ.(4)

p=2 $c_{n+1} = c_n - \lambda_2 y_n \cdot z_n (|z_n|^2 - R_2)$ wherein $R_2 = \frac{E(|a_m|^4)}{E(|a_m|^2)}$ Equ.(5)

where c_n is a vector of the tap weights, λ_1 and λ_2 are
adjusting coefficients, y_n is an input signal, z_n is an
equalized signal (output of the equalizer), R_p is a positive
20   number determined by the pattern of the constellation
diagram, and a_m represents the exact coordinates on the
constellation   diagram.   Simulations   show   that   the
convergence of tap weights is faster and more accurate when
using the second order (P=2) cost function rather than the
25   first order (p=1) cost function.

Drawbacks of the CMA algorithm include the performance
of convergence for a Quadrature Amplitude Modulation (QAM)
signal degrading with the number of valid coordinates on the
constellation diagram. Figure 2 shows the output of a CMA
5   equalizer for a signal modulated by 896-QAM. There are 896
valid coordinates on the constellation diagram for a 896-QAM
signal. As shown in Figure 2, detecting each constellation
point on the constellation diagram becomes impossible if the
number of valid coordinates is too large.

10   **SUMMARY OF THE INVENTION**

The object of the present invention is to provide an
adaptive multi-modulus equalization method for a blind
equalizer which overcomes the disadvantages of existing
methods.

15   The adaptive multi-modulus equalization method for an
adaptive equalizer proposed in the present invention
comprises the following steps. First, an input signal is
stored and initial values for a plurality of equalizer
coefficients (tap weights) are set. Then, the input signal
20   is passed into the adaptive equalizer to generate an
equalized signal. A cost is then calculated by substituting
the equalized signal into a preset cost function. The cost
is used to derive an adjustment amount for updating the tap
weights of the adaptive equalizer.   These steps are
25   identical to the CMA algorithm in the prior to allow the tap
weights to be updated for cost reduction.   The adaptive
equalizer switches to implementation of a multi-modulus
algorithm (MMA) rather than the CMA algorithm when the cost
is reduced to reach a first threshold.   The MMA algorithm

divides the constellation diagram into a plurality of regions, and the number of regions increases when switching to the subsequent stage. A key feature of the present invention is that there are stages with different numbers of

5   regions. Modulus of each region for the subsequent stage is determined by statistical analysis of the equalized signal generated by the equalizer.

The stored signal passes through the adaptive equalizer again when the equalizer switches to the MMA algorithm.

10   According to the MMA algorithm, the cost is now calculated regarding the equalized signal generated by the adaptive equalizer, as well as moduli of the corresponding regions. Then, an adjustment amount is derived by the cost, and is used to update the tap weights of the equalizer. The tap

15   weights are continuously adjusted to reduce the cost. The adaptive equalizer switches to the subsequent stage when the cost reaches the threshold of the current stage. The thresholds are preset to switch to subsequent stages. The adaptive equalizer repeats the process of convergence by

20   continuously updating the tap weights, reducing the cost, and switching to the subsequent stage until a desired value for the cost function output (cost) is reached. When the cost reaches the desired value, the number of regions and tap weights are fixed and the received signal is equalized

25   thereby.

These and other objectives of the present invention will be obvious to those of ordinary skill in the art after reading the following detailed description of the preferred embodiment, illustrated in the various Figures and drawings.

BRIEF DESCRIPTION OF THE DRAWINGS

The present invention will become more fully understood from the detailed description given herein and the accompanying drawings, given by way of illustration only and 5 thus not intended to be limitative of the present invention.

Fig. 1 is a diagram illustrating the ISI effect on a signal.

Fig. 2 is a constellation diagram illustrating an output of a CMA equalizer when receiving an 896-QAM signal.

10 Fig. 3 is a graph comparing the performance and the convergence rate between a CMA algorithm and an adaptive MMA algorithm.

Fig. 4 is a diagram illustrating a cost function of the CMA algorithm.

15 Fig. 5A and Fig. 5B are graphs illustrating two conditions of reducing the cost function using the steepest gradient descent method.

Fig. 6 is a constellation diagram illustrating regions and moduli in the MMA algorithm.

20 Fig. 7 illustrates the structural change of the constellation diagram when switching to the subsequent stage according to the adaptive MMA algorithm.

Fig. 8 is a block diagram illustrating a configuration of an adaptive equalizer of the present invention.

25 Fig. 9 is a block diagram illustrating a configuration of an equalizing system using the CMA algorithm.

Fig. 10 is a block diagram illustrating a configuration of an equalizing system using the adaptive MMA algorithm.

Fig. 11 is a flowchart illustrating an adaptive MMA equalization method according to the embodiment of the present invention.

Fig. 12 is a constellation diagram illustrating an output of a CMA equalizer when receiving an 896-QAM signal.

Fig. 13 is a constellation diagram illustrating an output of an adaptive MMA equalizer when receiving an 896-QAM signal.

Fig. 14 is a constellation diagram illustrating an output of a CMA equalizer when receiving a 448-QAM signal.

Fig. 15 is a constellation diagram illustrating an output of an adaptive MMA equalizer when receiving a 448-QAM signal.

Fig. 16A is a diagram illustrating an 112-QAM signal after non-linear transformation and phase recovery.

Fig. 16B is a diagram illustrating an 896-QAM signal after non-linear transformation and phase recovery.

Fig. 17A is a constellation diagram illustrating a signal received before passing through the decision feedback equalizer.

Fig. 17B is a constellation diagram illustrating a signal received after passing through the decision feedback equalizer.

DETAILED DESCRIPTION OF THE INVENTION

Figure 3 shows a convergence rate for the adaptive MMA algorithm exceeds the CMA algorithm as the adaptive MMA algorithm has multiple stages. By switching to subsequent stages, the convergence rate is increased because the constellation diagram is divided into more regions.   A

distance between any valid coordinate and its corresponding
modulus is decreased by allowing more regions on the
constellation diagram.   Costs can be reduced more
effectively by minimizing this distance, as explained later
5   in detail. The adaptive MMA algorithm solves the problem of
unacceptable convergence performance for QAM signals with
large numbers of valid coordinates on the constellation
diagram.

Figure 4 is a constellation diagram illustrating a cost
10   function. According to the CMA algorithm, all constellation
points fall in an area bounded by a circle with a centre o
and a radius R2'. The area of the circle is referred to as
region and the circumference of the circle as modulus on the
constellation diagram.   The radius R2' is limited by an
15   automatic gain control of the receiver, which limits the
space between constellation points. As shown in Figure 4,
an equalized signal falls into point A, and a closest valid
coordinate on the constellation point is at point B.   The
intention of convergence is to close the distance d between
20   point A and point B. An acceptable value for convergence of
d decreases with increased valid coordinates on the
constellation diagram. Controlling the distance d to within
the acceptable limit helps identify the corresponding
coordinates for every constellation point. The cost (output
25   of a cost function) of point A is approximately the sum of d
and c (d+c), where c is the distance between the closest
valid coordinate (point B) and the modulus and c is a
constant. Reducing the cost also reduces the distance d,
and the cost can be reduced by implementing the steepest
30   gradient descent method. The problem with the CMA algorithm

is that c is a constant and cannot be reduced any further, hence the distance d becomes relatively small during reduction of the cost. The steepest gradient descent method cannot execute effectively and efficiently when the ratio of

5   d to the cost becomes too small.

Figures 5A and 5B are graphs illustrating two types of condition for reducing the cost using the steepest gradient descent method. Figure 5A demonstrates a condition in which c is larger than d, where Figure 5B reduces the proportion

10   of c. The two graphs show that the steepest gradient descent method effectively decreases the distance d in Figure 5B as the gradient therein increases in steepness. Thereby, decreasing the proportion of c can speed the convergence of the cost.

15   The cost function is given by:

$$D^{(2)} = E\left[\left(|z_n|^2 - R_2\right)^2\right]$$ Equ. (6)

$$\text{where } \left(R_2 - |z_n|^2\right)^2 = \left(\sqrt{R_2} + |z_n|\right)^2 \cdot \left(\sqrt{R_2} - |z_n|\right)^2$$ Equ. (7)

$$(R_2 - |z_n|^2)^2 \approx \left(\sqrt{R_2} + |z_n|\right)^2 \cdot \left[\left(\sqrt{R_2} - |S_n|\right)^2 + \left(|S_n| - |z_n|\right)^2\right]$$ Equ. (8)

where S_n is a valid coordinate on the constellation

20   diagram, and z_n is a coordinate of an equalized signal generated by the equalizer. Equation (8) approximates the cost function, wherein the second operand of the equation represents the sum of two distances. $\left(\sqrt{R_2} + |S_n|\right)^2$ represents the distance between the modulus and the valid coordinate

25   closest to the coordinate of the equalized signal, is the

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distance c mentioned above. $\left(|S_n|-|z_n|\right)^2$ is the distance
between the coordinate of the equalized signal and the
closest valid coordinate, the distance d mentioned above.
To ensure proper convergence of the equalized signal, the
5   second operand of the equation must be reduced to an
acceptable value.   The acceptable value depends on the
number of valid coordinates on the constellation diagram,
such that increased valid coordinates implies stricter
acceptable value.   According to Figure 5A and 5B, the
10   steepest gradient descent method is more effective if the

distance c, $\left(\sqrt{R_2}+|S_n|\right)^2$ in equation (8) can be reduced.

Multi-modulus technique proposed in the present invention
reduces the distance c by dividing the constellation
diagram into regions as shown in Figure 6.

15   Figure 6 is a constellation diagram illustrating
regions and moduli in the multi-modulus algorithm (MMA).
As shown in Figure 6, the regions are concentric circles on
the   constellation   diagram,   each   region   having   a
corresponding modulus.   In comparison with conventional
20   methods using the same modulus for cost calculation, a
different modulus is used when the equalized signal falls
into a different region.   The MMA algorithm reduces the
distance between the modulus and the valid coordinate
(distance c) by replacement with a modulus closer to the
25   coordinate of the equalized signal.   For example, the
modulus closest to the centre of the circle is used when
the equalized signal falls into region 1, and the largest
modulus is used when the equalized signal falls into region

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3. The distance c, the constant part of the cost function is hence reduced, speeding the process of convergence.

In the embodiment of the present invention, the number of regions gradually increases at each stage during convergence. Figure 7 is an example of switching from one stage to the subsequent stage, in which the number of regions increases from two to three. Initially the constellation diagram contains only one region, as in the case of CMA algorithm. The first stage uses the CMA algorithm to estimate the convergence result. The modulus of each region for the subsequent stage may be estimated according to the following equation by analyzing the coordinates of the equalized signal in each region.

$$R_2 = \frac{E\left[|Z_n|^4\right]}{E\left[|Z_n|^2\right]} \approx \frac{E\left[|a_m|^4\right]}{E\left[|a_m|^2\right]} \qquad \text{Equ.(9)}$$

where z_n represents the coordinates of the equalized signal. The modulus of each region for the subsequent stage is obtained by statistical analysis while converging the signal. The adaptive MMA algorithm has several thresholds for determining when to switch to the subsequent stage. For example, the adaptive MMA algorithm switches to the second stage with two regions when the cost reaches the first threshold, and to the third stage with three regions when the cost reaches the second threshold. The adaptive equalizer has a faster convergence rate when switching to the subsequent stage with more regions on the constellation diagram, whereas it is also possible to switch back to the previous stage with fewer regions if the cost increases during convergence rather than decreasing. Switching back

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to the previous stage avoids bottlenecks during convergence at a particular stage. A possible cause of bottleneck is the equalized signal continues to fall into the region boundaries.

5   The equation for calculating the modulus relates to the coordinates on the constellation diagram. The embodiment of the present invention estimates the modulus for each region according to the equalized signal. This ensures that the estimated moduli will not be affected by the number of
10   symbols transmitted in the signal.

     Figure 8 is a block diagram illustrating an equalizing system for an adaptive equalizer using both the CMA algorithm and the adaptive MMA algorithm. The configuration of the equalizing system comprises an equalizer 10, a
15   multiplexer 20, a CMA tap-weight generator 30, and an adaptive MMA tap-weight generator 40. The equalizer 10 equalizes an input signal 50 according to its coefficients (tap weights). The coefficients of the equalizer 10 are updated according to tap weights generated by either the CMA
20   tap-weight generator 30 or the adaptive MMA tap-weight generator. The multiplexer 20 first selects the tap weights generated by the CMA tap-weight generator 30 until the cost reaches the first threshold as discussed before.

     Figures 9 and 10 show the configurations of the CMA
25   tap-weight generator 30 and the adaptive MMA tap-weight generator 40 respectively. As shown in Figure 9, the equalizer 10 receives new tap weights from a coefficient generator 38. The coefficient generator 38 calculates the new tap weights according to the input signal 50, an
30   equalized signal 60 from the equalizer 10, and a

corresponding modulus generated by a constant modulus
generator 35.   As shown in Figure 10, the equalizer 10
receives new tap weights from the coefficient generator 48.
The coefficient generator 48 calculates the new tap weights
5   according to the input signal 50, the equalized signal 60,
and an output signal of a modulus estimator 42.   The modulus
estimator 42 gathers information from the equalizer 10, and
a modulus selector 45, and calculates modulus of each region
for the subsequent stage.

10   Figure 11 is a flowchart illustrating the algorithm of
the equalization method according to the embodiment of the
present invention.   Step S101 receives an input signal for
equalization, and step S102 stores the input signal.   The
input signal then passes through the adaptive equalizer of
15   the present invention in step S103.   The coefficients (tap
weights) of the adaptive equalizer are first set to initial
values, and will be modified later.   Step S104 determines if
the cost (output of the cost function) is reduced to a first
threshold.   Step S105 switches from the CMA algorithm to the
20   MMA algorithm when the cost is less than the first
threshold, otherwise, the equalizer continues to update the
tap weights using the CMA algorithm.   Step S110 calculates
the cost according to the CMA algorithm, and step S111 sends
the newly updated tap weights to the adaptive equalizer, and
25   continues updating the tap weights using the CMA algorithm
until the cost is below the first threshold in step S104.
Step S112 computes the equalized signal according to the CMA
algorithm, and then step S124 estimates the moduli for the
subsequent stage by statistical analysis of the equalized
30   signal.   When the adaptive equalizer starts to use the MMA

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algorithm, step S120 determines the corresponding region for each symbol in the input signal. The cost is now estimated using the MMA algorithm, and the moduli calculated in step S124 are used to calculate the cost in step S121. Step S122
5 updates the tap weights, and returns to step S103 to repeat the processes of updating the tag weights. When switching to a new stage, step S121 uses the moduli estimated in step S124 to calculate the cost. Switching between stages is determined by step S125, only switching to the subsequent
10 stage when the cost is below a threshold corresponding to the current stage.

Figure 12 is a constellation diagram illustrating an output of a CMA equalizer when receiving an 896-QAM signal, where Figure 13 is a constellation diagram illustrating an
15 output of an adaptive MMA equalizer when receiving an 896-QAM signal. It is obvious that the convergence performed by the adaptive MMA equalizer is better than the conventional CMA equalizer. Figures 14 and 15 also show that the constellation points of a 448-QAM signal on the
20 constellation diagram converging by means of the adaptive MMA equalizer as shown in Figure 15 are clearly separated, and hence perform better than the CMA equalizer shown in Figure 14.

The adaptive equalizer implementing both the CMA
25 algorithm and the MMA algorithm in the present invention can also receive non-linear coded signal. Non-linear encoder often multiply the signal strength with a factor to increase the distance between constellation points located further away, and decrease the distance between constellation points
30 located closer to the centre of the constellation diagram.

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By multiplying the factor, the quantization error in the telephone network according to μ-law and A-law is reduced. By the relationship between detected clusters and original coordinates, a transformation formula, referred to as non-linear transformation, can scale these equalized signals to their original magnitude. An example of the non-linear equation $y = ax^2 + bx + c$ is a parabola, where y is an ideal receiving power, and x is a received power. The coefficients a, b, and c are found by minimizing the error function $S = \sum \left[y_i - ax^2 + bx + c \right]$.

Phase difference is induced in the received signal when the receiving end is not synchronized with the transmitting end. Neither CMA nor MMA equalizer can compensate for this phase difference. The amplitude of the signal is accurate after non-linear transformation, but there is a need to rotate the constellation diagram to compensate for the phase shift. Multiplying the received signal by an unit vector rotates the signal by the smallest angle to upright position, such as (a+jb), where $a^2 + b^2 = 1$, to minimize the error function $S = \sum [(p_i + jq_i)(a_i + jb_i) - (u_i + v_i)]^2$, where $(p_i + jq_i)$ represents coordinates of an input signal after non-linear transformation, and $(u_i + v_i)$ represents valid coordinates on the constellation diagram. Figure 16A illustrates a 112-QAM signal after non-linear transformation and phase recovery, and Figure 16B illustrates a 896-QAM signal after non-linear transformation and phase recovery. The signals in these two

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Figures are converged and located very close to the valid coordinates on the constellation diagram.

Signals passing through the adaptive MMA equalizer still have some ISI, which is then regarded as noise in the

5   signal.   The remaining ISI can be removed by a decision feedback equalizer comprising a feed forward filter and a feedback filter after non-linear transformation and phase recovery.   Figure   17A   is   a   constellation   diagram illustrating a signal received before passing through the

10   decision   feedback   equalizer,   and   Figure   17B   is   a constellation diagram illustrating the received signal after passing through the decision feedback equalizer.   Figures 17A and 17B show the decision feedback equalizer effectively removing most of the remaining ISI from the signal.

15   There are many advantages to the adaptive MMA equalizer disclosed in the present invention.   The number of regions increases in each stage, and the rate of convergence of the tap weights is improved by more regions on the constellation diagram.   The convergence performs well when the received

20   signal is modulated to a constellation diagram with a large number of valid coordinates.   The adaptive MMA equalizer is suitable for all kinds of QAM signals as the number of regions and modulus of each region are adjustable.   The adaptive MMA equalizer is also applicable for signals with

25   variable distances between constellation points on the constellation diagram.   For example, V.34 protocol includes multiplying the distances between constellation points with a non-linear function.   The variable distances can be recovered by adjusting the corresponding modulus during a

30   non-linear transformation.

The foregoing descriptions of the embodiment of this invention has been presented for purposes of illustration and description. Obvious modifications or variations are possible in light of the above teaching. The embodiment was chosen and described to provide the best illustration of the principles of this invention and its practical application to thereby enable those skilled in the art to utilize the invention in varies applications. All such modifications and variations are within the scope of the present invention as determined by the appended claims when interpreted in accordance with the breadth to which they are fairly, legally, and equitably entitled.